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Probing Anomalous Top-Couplings through the Final Lepton Angular and Energy Distributions at Polarized NLC *

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ABSTRACT

The angular and energy distributions of the final lepton in $e\bar{e} \rightarrow t\bar{t} \rightarrow \ell^\pm X$ at next linear colliders (NLC) are analyzed a model-independent way for arbitrary longitudinal beam polarizations as sensitive tests of possible anomalous top-quark couplings. The angular-energy distribution is expressed as a combination of independent functions of the angle and the energy, where each anomalous parameter is the coefficient of an individual term. Every parameter could be thereby determined simultaneously via the optimal-observable procedure. On the other hand, anomalous tbW couplings totally decouple from the angular distribution, which enables us to study $t\bar{t}\gamma/Z$ couplings exclusively.

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1. Introduction

The discovery of the top-quark [1] completed the fermion list required in the standard EW theory (SM). However it is still an open question whether this quark interacts with the others the standard way or there exists any new-physics contribution to its couplings. It decays immediately after being produced because of the huge mass. Therefore this process is not influenced by any hadronization effects and consequently the decay products are expected to tell us a lot about parent top property.

Next linear colliders (NLC) of $e\bar{e}$ will give us fruitful data on the top through $e\bar{e} \rightarrow t\bar{t}$. In particular the final lepton(s) produced in its semileptonic decay(s) turns out to carry useful information of the top-quark couplings [2]. Indeed many authors have worked on this subject (see the reference list of Ref.[3]), and we also have tackled them over the past several years.

Here I would like to show some of the results of our latest model-independent analyses of the lepton distributions for arbitrary longitudinal beam polarizations [3], where we have assumed the most general anomalous couplings both in the production and decay vertices in contrast to most of the existing works.^{#1} What we actually studied are the lepton angular-energy distribution and the angular distribution, both of which would enable us to perform interesting tests of the top-quark couplings.

2. Framework

We can represent the most general covariant $t\bar{t}$ couplings to the photon and Z boson as^{#2}

$$\Gamma_{vt\bar{t}}^\mu = \frac{g}{2} \bar{u}(p_t) \left[\gamma^\mu (A_v - B_v \gamma_5) + \frac{(p_t - p_{\bar{t}})^\mu}{2m_t} (C_v - D_v \gamma_5) \right] v(p_{\bar{t}}) \quad (1)$$

^{#1}I would like to focus on the final lepton although we studied also the b distributions in $e\bar{e} \rightarrow t\bar{t} \rightarrow b(\bar{b})X$ in [3].

^{#2}Throughout this report, I use simplified expressions. Here, for example, A_v means $A_v + \delta A_v$ in our original papers.

in the $m_e = 0$ limit, where g denotes the $SU(2)$ gauge coupling constant, $v = \gamma, Z$ and

$$\begin{aligned} A_\gamma &= 4 \sin \theta_W / 3, \quad B_\gamma = 0, \quad A_Z = (1 - 8 \sin^2 \theta_W / 3) / (2 \cos \theta_W), \\ B_Z &= 1 / (2 \cos \theta_W), \quad C_\gamma = D_\gamma = C_Z = D_Z = 0 \end{aligned}$$

within the SM. Among the above form factors, $A_{\gamma,Z}$, $B_{\gamma,Z}$ and $C_{\gamma,Z}$ are parameterizing CP -conserving interactions, while $D_{\gamma,Z}$ is CP -violating one.

On the other hand, we adopted the following parameterization of the tbW vertex suitable for the $t \rightarrow W^+ b$ and $\bar{t} \rightarrow W^- \bar{b}$ decays:

$$\Gamma_{Wtb}^\mu = -\frac{g}{\sqrt{2}} \bar{u}(p_b) \left[\gamma^\mu (f_1^L P_L + f_1^R P_R) - \frac{i\sigma^{\mu\nu} k_\nu}{M_W} (f_2^L P_L + f_2^R P_R) \right] u(p_t), \quad (2)$$

$$\bar{\Gamma}_{Wtb}^\mu = -\frac{g}{\sqrt{2}} \bar{v}(p_{\bar{t}}) \left[\gamma^\mu (\bar{f}_1^L P_L + \bar{f}_1^R P_R) - \frac{i\sigma^{\mu\nu} k_\nu}{M_W} (\bar{f}_2^L P_L + \bar{f}_2^R P_R) \right] v(p_{\bar{b}}), \quad (3)$$

where k is the W -boson momentum, $P_{L/R} = (1 \mp \gamma_5)/2$ and

$$f_1^L = \bar{f}_1^L = 1, \quad f_1^R = \bar{f}_1^R = f_2^{L,R} = \bar{f}_2^{L,R} = 0$$

within the SM. This is also the most general form as long as we treat W as an on-mass-shell particle, which is indeed a good approximation. It is worth mentioning that these form factors satisfy the following relations [4]:

$$f_1^{L,R} = \pm \bar{f}_1^{L,R}, \quad f_2^{L,R} = \pm \bar{f}_2^{R,L}, \quad (4)$$

where upper (lower) signs are those for CP -conserving (-violating) contributions.

For the initial beam-polarization we used the following convention:

$$P_{e^\mp} = \pm [N(e^\mp, +1) - N(e^\mp, -1)] / [N(e^\mp, +1) + N(e^\mp, -1)], \quad (5)$$

where $N(e^\pm, h)$ is the number of e^\pm with helicity h in each beam. Note that P_{e^+} is defined with $+$ sign instead of $-$ in some literature.

3. Angular-energy distributions

After some calculations, we arrived at the following angular-energy distribution of the final lepton ℓ^+ :

$$\frac{d^2\sigma}{dx d\cos\theta} = \frac{3\pi\beta\alpha^2}{2s} B \left[S^{(0)}(x, \theta) \right]$$

$$\begin{aligned}
& + \sum_{v=\gamma, Z} [\text{Re}(\delta A_v) \mathcal{F}_{A_v}(x, \theta) + \text{Re}(\delta B_v) \mathcal{F}_{B_v}(x, \theta) \\
& \quad + \text{Re}(\delta C_v) \mathcal{F}_{C_v}(x, \theta) + \text{Re}(\delta D_v) \mathcal{F}_{D_v}(x, \theta)] \\
& + \text{Re}(f_2^R) \mathcal{F}_{2R}(x, \theta)], \tag{6}
\end{aligned}$$

where $\beta (\equiv \sqrt{1 - 4m_t^2/s})$ is the top-quark velocity, B denotes the appropriate branching fraction ($=0.22$ for e/μ), x means the normalized energy of ℓ defined in terms of its energy E as

$$x \equiv \frac{2E}{m_t} \sqrt{\frac{1-\beta}{1+\beta}},$$

θ is the angle between the e^- beam direction and the ℓ momentum (Fig.1), all in the e^+e^- CM frame, $S^{(0)}$ is the SM contribution, $\delta A_v \sim \delta D_v$ express non-SM part of $A_v \sim D_v$ (i.e., δB_γ , δC_v and δD_v are equal to B_γ , C_v and D_v respectively), and \mathcal{F} are analytically-expressed functions of x and θ , which are independent of each other. We neglected all $|\text{non-SM term}|^2$ as a reasonable assumption (see [5]). Replacing δD_v , f_2^R and $\cos \theta$ with $-\delta D_v$, \bar{f}_2^L and $-\cos \theta$ gives the ℓ^- distribution.

Equation (6) can be re-expressed as

$$\frac{d^2\sigma}{dx d\cos\theta} = \frac{3\pi\beta\alpha^2}{2s} B \left[\Theta_0(x) + \cos\theta \Theta_1(x) + \cos^2\theta \Theta_2(x) \right]. \tag{7}$$

This form directly leads to the angular distribution for ℓ through the integration over x :

$$\frac{d\sigma}{d\cos\theta} = \int_{x_-}^{x_+} dx \frac{d^2\sigma}{dx d\cos\theta} = \frac{3\pi\beta\alpha^2}{2s} B \left(\Omega_0 + \Omega_1 \cos\theta + \Omega_2 \cos^2\theta \right), \tag{8}$$

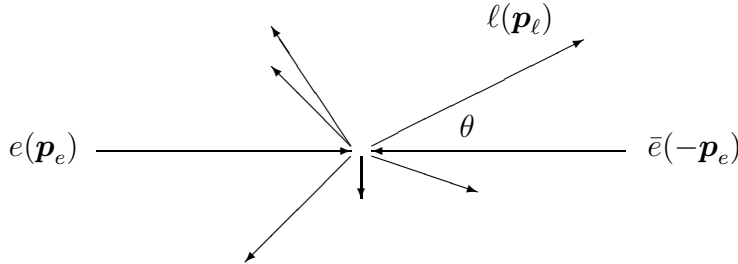


Figure 1: Scattering angle of the final lepton ℓ

where $\Omega_{0,1,2} \equiv \int_{x_-}^{x_+} dx \Theta_{0,1,2}(x)$ and x_{\pm} define the kinematical range of x .

Surprisingly enough, the non-SM decay part, i.e., f_2^R term completely disappears through this x integration, and the angular distribution depends only on the whole production vertex plus the SM decay vertex [3, 6]. This never happens in the final b -quark distribution.

4. Numerical analyses

First, we could determine $\delta A_v \sim \delta D_v$ and f_2^R simultaneously using the angular-energy distribution (6) via the optimal-observable procedure [7], since these anomalous parameters are all coefficients of independent functions. In the second paper of Ref.[3], we explored the best e^{\pm} polarizations which minimize the expected statistical uncertainty (1σ) for each parameter.

Our results assuming the integrated luminosity $L = 500 \text{ fb}^{-1}$ and the lepton-detection efficiency $\epsilon = 60\%$ at $\sqrt{s} = 500 \text{ GeV}$ are

$$\begin{aligned}
\Delta[\text{Re}(\delta A_{\gamma})] &= 0.16 & \text{for } P_{e^-} = +0.7 & \text{and } P_{e^+} = +0.7, \\
\Delta[\text{Re}(\delta A_Z)] &= 0.07 & \text{for } P_{e^-} = +0.5 & \text{and } P_{e^+} = +0.4, \\
\Delta[\text{Re}(\delta B_{\gamma})] &= 0.09 & \text{for } P_{e^-} = +0.2 & \text{and } P_{e^+} = +0.2, \\
\Delta[\text{Re}(\delta B_Z)] &= 0.27 & \text{for } P_{e^-} = +0.4 & \text{and } P_{e^+} = +0.4, \\
\Delta[\text{Re}(\delta C_{\gamma})] &= 0.11 & \text{for } P_{e^-} = +0.1 & \text{and } P_{e^+} = 0.0, \\
\Delta[\text{Re}(\delta C_Z)] &= 1.11 & \text{for } P_{e^-} = +0.1 & \text{and } P_{e^+} = 0.0, \\
\Delta[\text{Re}(\delta D_{\gamma})] &= 0.08 & \text{for } P_{e^-} = +0.2 & \text{and } P_{e^+} = +0.1, \\
\Delta[\text{Re}(\delta D_Z)] &= 14.4 & \text{for } P_{e^-} = +0.2 & \text{and } P_{e^+} = +0.1, \\
\Delta[\text{Re}(f_2^R)] &= 0.01 & \text{for } P_{e^-} = -0.8 & \text{and } P_{e^+} = -0.8.
\end{aligned} \tag{9}$$

In spite of the large L , the precision does not seem so good. At first sight, readers might conclude that these results contradict, e.g., the results in [8] which give higher precision. It is however premature to draw such a conclusion. In [8], they varied just one parameter in one trial, while we varied all the parameters simultaneously, which is realistic if we have no other information. I confirmed that we get a result similar to theirs if we carry out an analysis the same way. This means that our results could be improved by any other statistically-independent data.

On the other hand, we can perform another interesting test via the angular

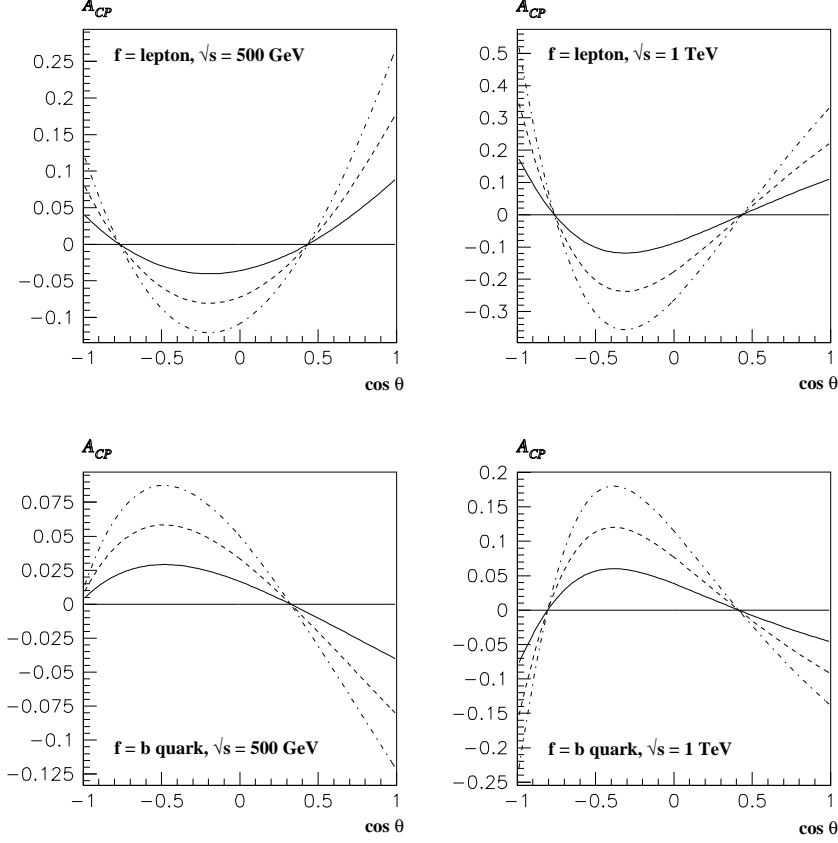


Figure 2: CP -violating asymmetry $\mathcal{A}_{CP}(\theta)$ as a function of $\cos \theta$ for leptonic and b -quark distributions for $\text{Re}(D_\gamma) = \text{Re}(D_Z) = \text{Re}(f_2^R - \bar{f}_2^L) = 0.1$ (solid line), 0.2 (dashed line), 0.3 (dash-dotted line) at $\sqrt{s} = 500$ GeV and 1 TeV collider energy. As mentioned in the main text, $\mathcal{A}_{CP}(\theta)$ for lepton only depends on $D_{\gamma,Z}$.

distribution. That is, asymmetries like

$$\mathcal{A}_{CP}(\theta) = \left[\frac{d\sigma^+(\theta)}{d\cos\theta} - \frac{d\sigma^-(\pi - \theta)}{d\cos\theta} \right] / \left[\frac{d\sigma^+(\theta)}{d\cos\theta} + \frac{d\sigma^-(\pi - \theta)}{d\cos\theta} \right] \quad (10)$$

or

$$\mathcal{A}_{CP} = \frac{\int_{-c_m}^0 d\cos\theta \frac{d\sigma^+(\theta)}{d\cos\theta} - \int_0^{+c_m} d\cos\theta \frac{d\sigma^-(\theta)}{d\cos\theta}}{\int_{-c_m}^0 d\cos\theta \frac{d\sigma^+(\theta)}{d\cos\theta} + \int_0^{+c_m} d\cos\theta \frac{d\sigma^-(\theta)}{d\cos\theta}}, \quad (11)$$

where $d\sigma^\pm$ are for ℓ^\pm respectively and c_m expresses an experimental angle cut, are a pure measure of the CP -violating anomalous $t\bar{t}\gamma/Z$ parameters (Fig.2).

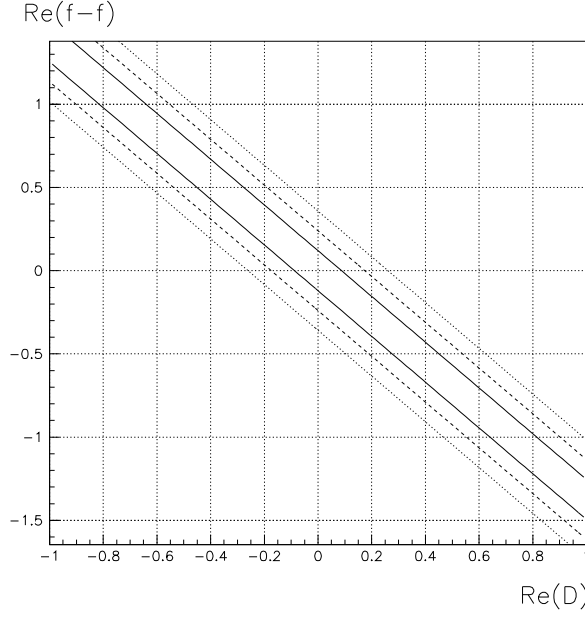


Figure 3: Parameter area which we can explore through the asymmetry $A_{\ell\ell}$ introduced in [9]. We can confirm this asymmetry to be non-zero at 1σ , 2σ and 3σ level when the parameters $\text{Re}(D_{\gamma,Z})$ and $\text{Re}(f_2^R - \bar{f}_2^L)$ are outside the two solid lines, dashed lines and dotted lines respectively. Unfortunately there is some area where two contributions from the production and decay vertices cancel each other and we get little information.

In Ref.[9], we introduced the following asymmetry

$$A_{\ell\ell} \equiv \frac{\iint_{x < \bar{x}} dx d\bar{x} \frac{d^2\sigma}{dx d\bar{x}} - \iint_{x > \bar{x}} dx d\bar{x} \frac{d^2\sigma}{dx d\bar{x}}}{\iint_{x < \bar{x}} dx d\bar{x} \frac{d^2\sigma}{dx d\bar{x}} + \iint_{x > \bar{x}} dx d\bar{x} \frac{d^2\sigma}{dx d\bar{x}}} \quad (12)$$

using the ℓ^\pm energy correlation $d^2\sigma/dxd\bar{x}$, where x and \bar{x} are the normalized energies of ℓ^+ and ℓ^- respectively. Generally this is also an asymmetry very sensitive to CP violation. However, when we have no luck and two contributions from the production and decay vertices cancel each other, we get little information as found in Fig.3. This comparison lightens the outstanding feature of $\mathcal{A}_{CP}(\theta)$ and \mathcal{A}_{CP} more clearly.

5. Summary

I showed here some results of our latest work on the angular and energy distributions of the lepton (e or μ) produced in $e\bar{e} \rightarrow t\bar{t} \rightarrow \ell^\pm X$. There the most general covariant forms were assumed both for the $t\bar{t}\gamma/Z$ and tbW couplings, which makes our analysis fully model-independent.

The angular-energy distribution $d^2\sigma/dxd\cos\theta$ could enable us to determine in principle all the anomalous parameters in the general $t\bar{t}\gamma/Z$ and tbW couplings simultaneously. Although extremely high luminosity is required to achieve good precision, it never means our analysis is impractical. We could get better precision when we have any other independent information on those anomalous parameters.

On the other hand, the angular distribution $d\sigma/d\cos\theta$ is completely free from the non-SM decay vertex. Therefore, once we catch any non-trivial signal of non-standard phenomena, it will be an indication of new-physics effects in $t\bar{t}\gamma/Z$ couplings. This is quite in contrast to asymmetries using the single or double energy distributions of $e\bar{e} \rightarrow t\bar{t} \rightarrow \ell^\pm X / \ell^+\ell^- X'$, where cancellation between the production and decay contributions could occur.

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